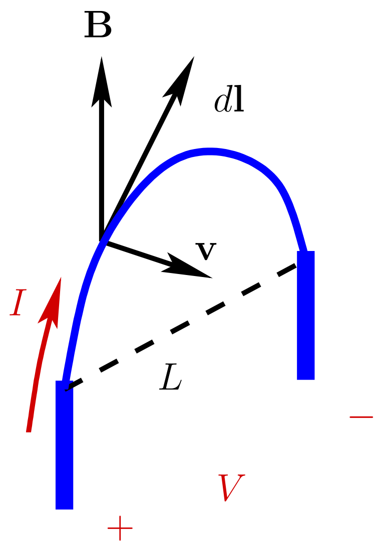
Vibrating wire thermometry in superfluid He^3-B

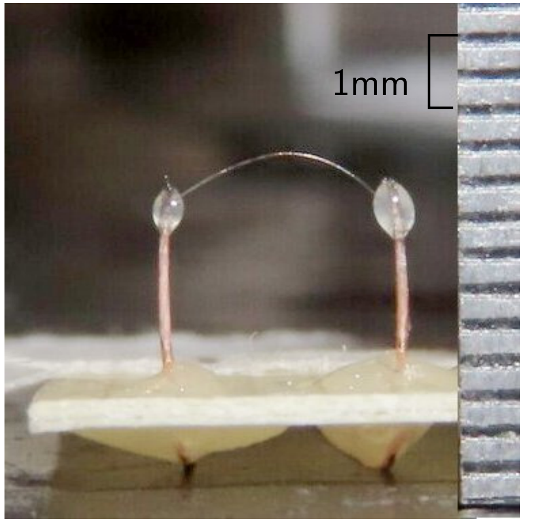
Vladislav Zavjalov

Royal Holloway, 10.5.2023

Vibrating wire in magnetic field

Wire loop in magnetic field $\mathbf{B}$ is driven by current I and moving with velocity $\mathbf{v}$.





Force acting on a piece of wire $d\mathbf{l}$:

$$d\mathbf{F} = I[d\mathbf{l} \times \mathbf{B}]$$

EMF voltage:

$$dV = \mathbf{v} \cdot [d\mathbf{l} \times \mathbf{B}]$$

By integrating along the wire we have total force and voltage:

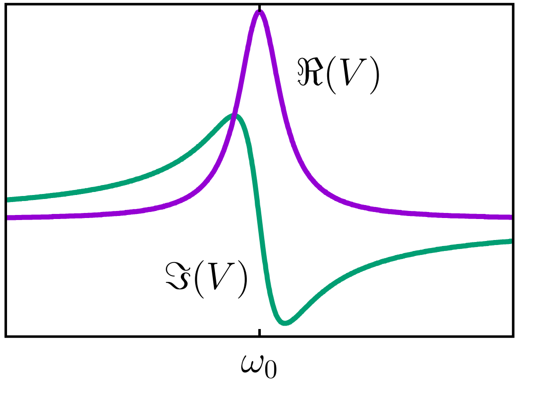
$F = ILB,\qquad V = vLB,\qquad$ where v is averege velocity

Equation of motion (linear damplng):

$$\ddot{x} = -\omega_0^2 x - \delta\,\dot{x} + \frac{ILB}{m_w}\cos\omega t$$

Using complex notation:

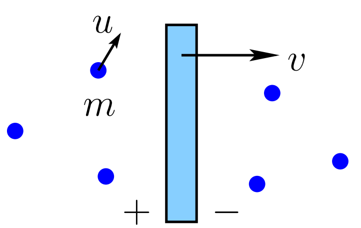
velocity: $\mathbf{v} = \dfrac{i\omega\,\mathbf{I}LB/m}{\omega_0^2 - \omega^2 + i\omega\delta}$ voltage: $\mathbf{V} = \dfrac{i\omega\,\mathbf{I}(LB)^2/m}{\omega_0^2 - \omega^2 + i\omega\delta}$



Non-linear damping in He^3-B

Classical gas

density n, mass m, average velocity u



Moving object

area A, velocity $v \ll u$

Amount of colliding particles at both sides per unit time: $\dot{N}_\pm = -nA(\langle u \rangle \pm v)$

Momentum transfered by a particle: $p_\pm = \pm 2m\langle u \rangle$

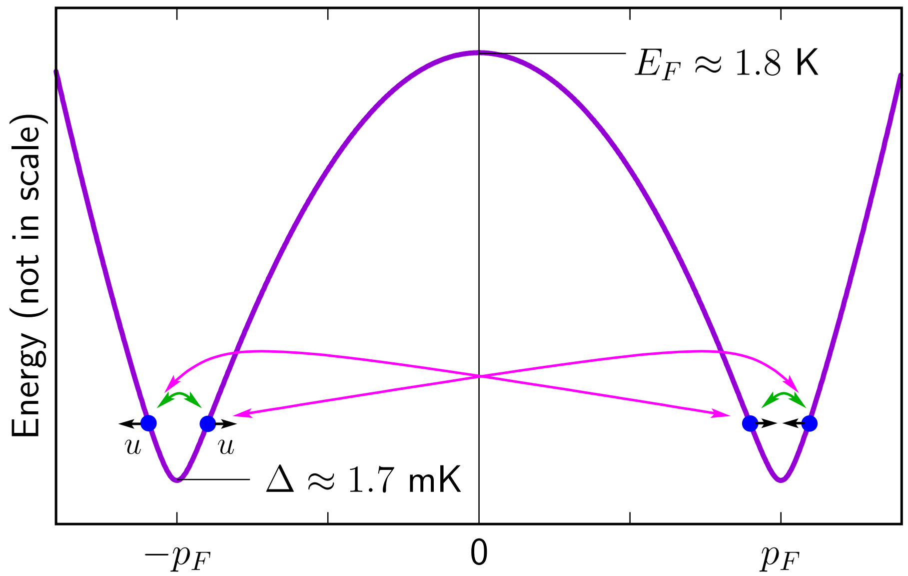
Force: $F = p_+\dot{N}_+ + p_-\dot{N}_- = -4nAm\langle u \rangle v$

Linear damping!

Superfluid ^{3}He-B

Energy spectrum of Bogoliubov quasiparticles:

$$E(p) = \sqrt{\left(\frac{p^2 - p_F^2}{2m}\right)^2 - \Delta^2} \qquad \text{Group velocity } u = \frac{dE}{dp}$$



Normal scattering ($u \to -u$, $p \to -p$)

Andreev scattering ($u \to -u$, $p \to p$)

Normal scttering can be prohibited by conservation of energy: quasiparticle can not loose energy and go below the gap

Force in a simple 1D model:

$$F = -F_0\, v_0\, (1 - \exp(-v/v_0)) \qquad v_0 = \frac{kT}{p_F} \qquad F_0 = Ap_F^2 v_F N(0) \exp\left(-\frac{\Delta}{kT}\right)$$

Non-linear motion of the wire

Non-linear equation of motion:

$$\ddot{x} = -\omega_0^2 x - \delta \dot{x} + \frac{ILB}{m_w} \cos\omega t$$

$\delta \dot{x} \rightarrow \delta v_0 f(\dot{x}/v_0)$ $f(x) \approx x$ at $x \to 0$

1D scattering model: $f(x) = \text{sign}(x)(1 - \exp(-|x|))$

After using van del Pol transformation and averaging over period:

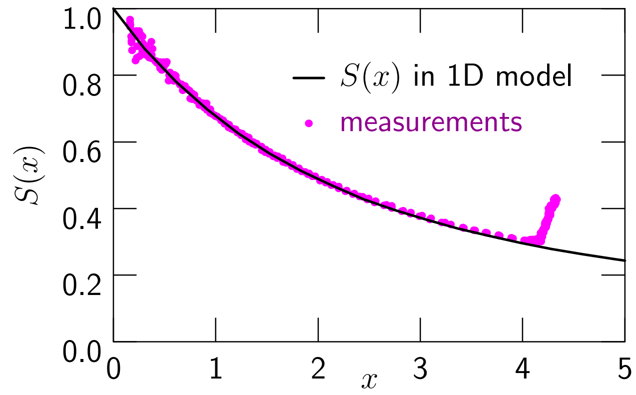
complex velocity: $\mathbf{v} = \dfrac{i\omega\, \mathbf{I} LB/m}{\omega_0^2 - \omega^2 + i\omega\ S(|v|/v_0)\delta}$ voltage: $\mathbf{V} = \dfrac{i\omega\, \mathbf{I} (LB)^2/m}{\omega_0^2 - \omega^2 + i\omega\ S(|V|/V_0)\delta}$

$$S(x) = -\frac{2}{x}\int_0^{2\pi} f(-x\sin(\omega t)\ \sin\omega t \frac{d(\omega t)}{2\pi}$$

If $f(x) = x + a\,\text{sign}(x)\,x^2$ then $S(x) = 1 + \dfrac{8a}{3\pi} x$

for 1D scattering model: $S(x) = \dfrac{2}{x}\left(I_1(x) - L_{-1}(x) + \dfrac{2}{\pi}\right)$,

where $I_n x$ is modified Bessel function of first kind and $L_n(x)$ is modified Struve function.



Measuring function S

Tracking mode:

1. measure resonance by sweeping frequency at constant drive current I_0

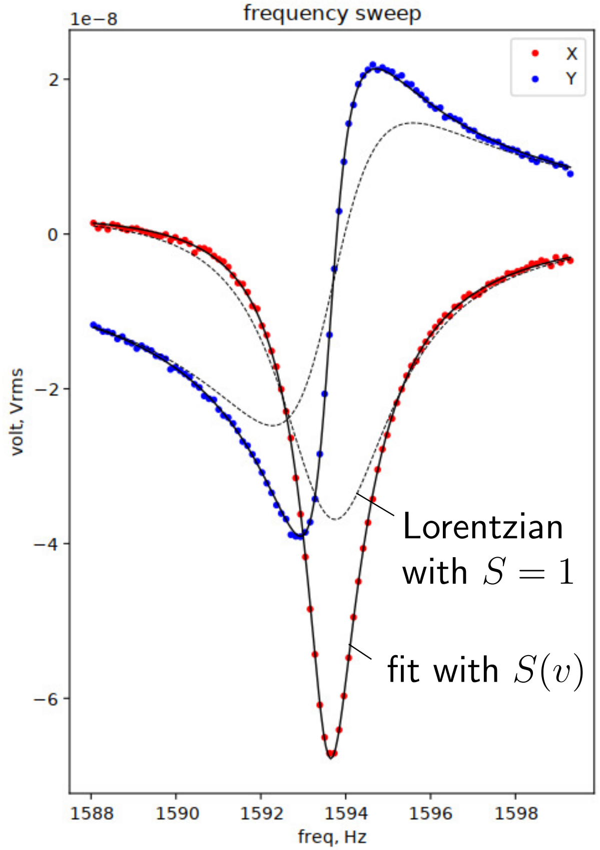
   $$\mathbf{V} = \frac{\mathbf{a}\, I_0\, i\omega}{\omega_0^2 - \omega^2 + i\omega S(V)\delta} + \mathbf{b}\, I_0$$

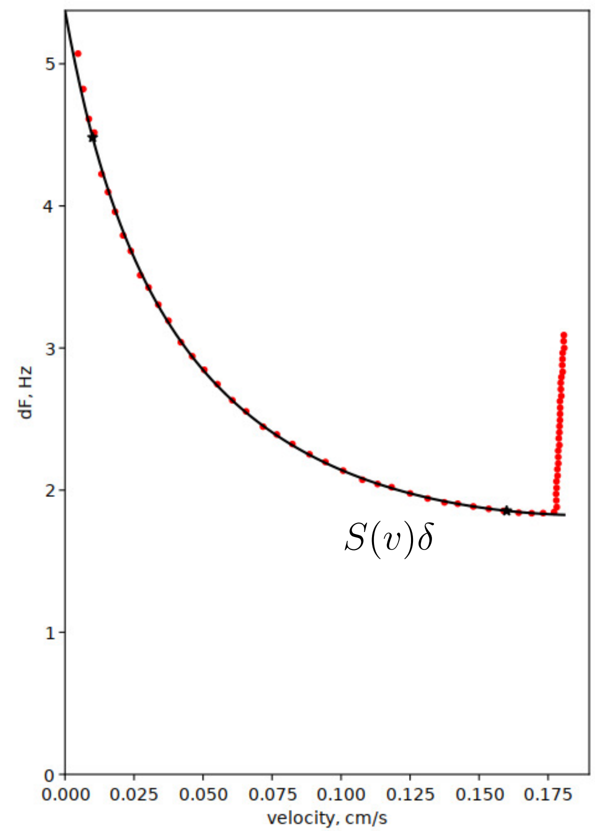
2. Find $\mathbf{a}$ and $\mathbf{b}$

3. Measure voltage as a function of drive at constant frequency, solve for S.

   $$S(V)\delta = \Re\left(\frac{\mathbf{a}\, I}{V - \mathbf{b}\, I}\right)$$

4. Iterate from step 1 with obtained function S

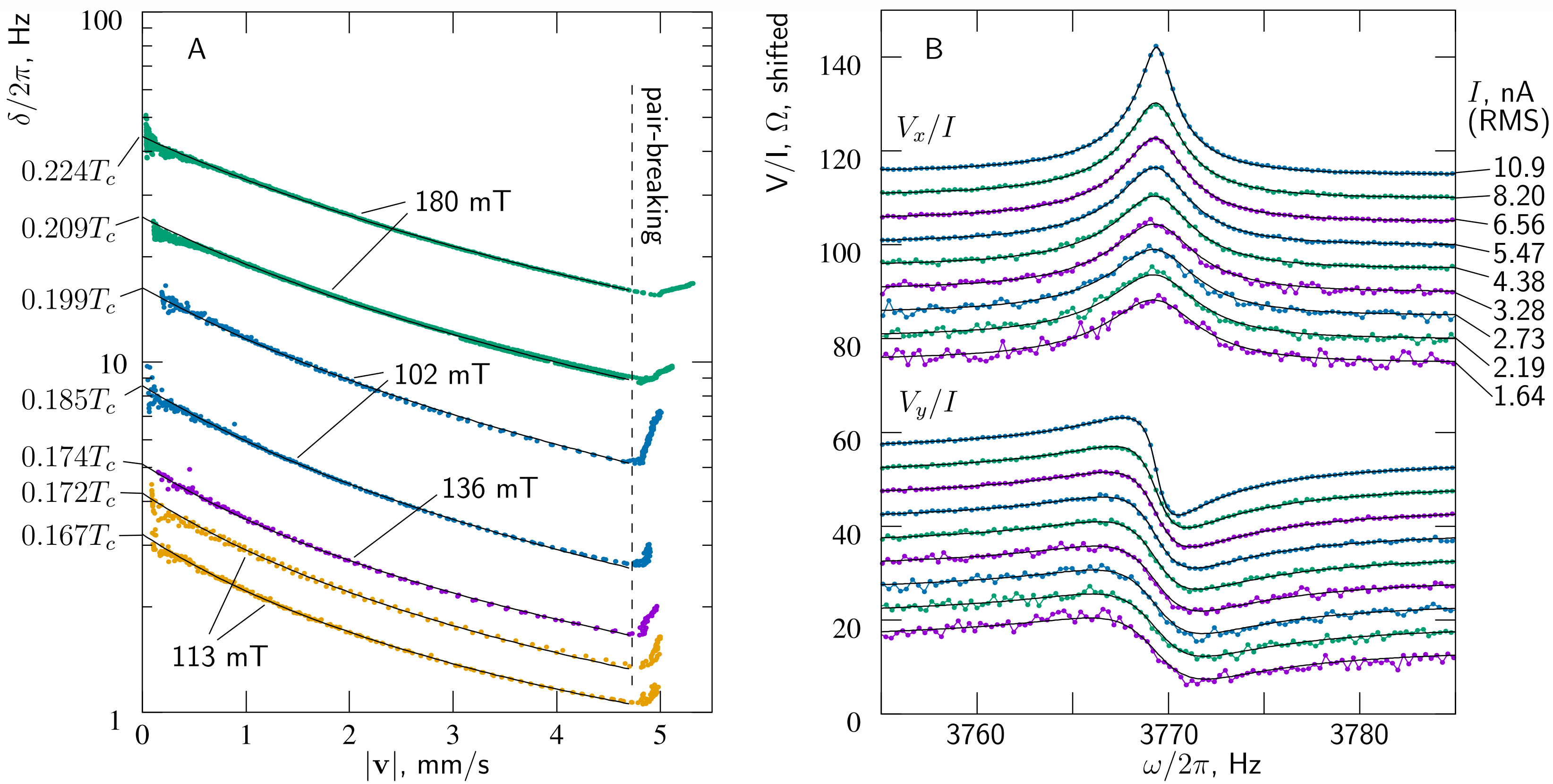




Ultra Low Temperature

Lancaster University

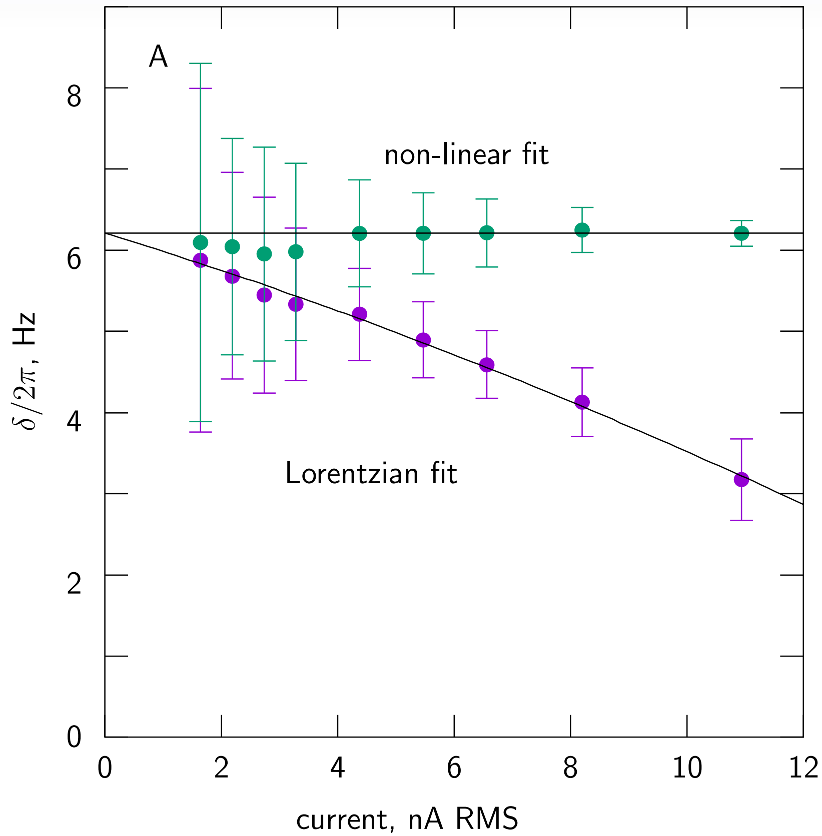
Building model for function S



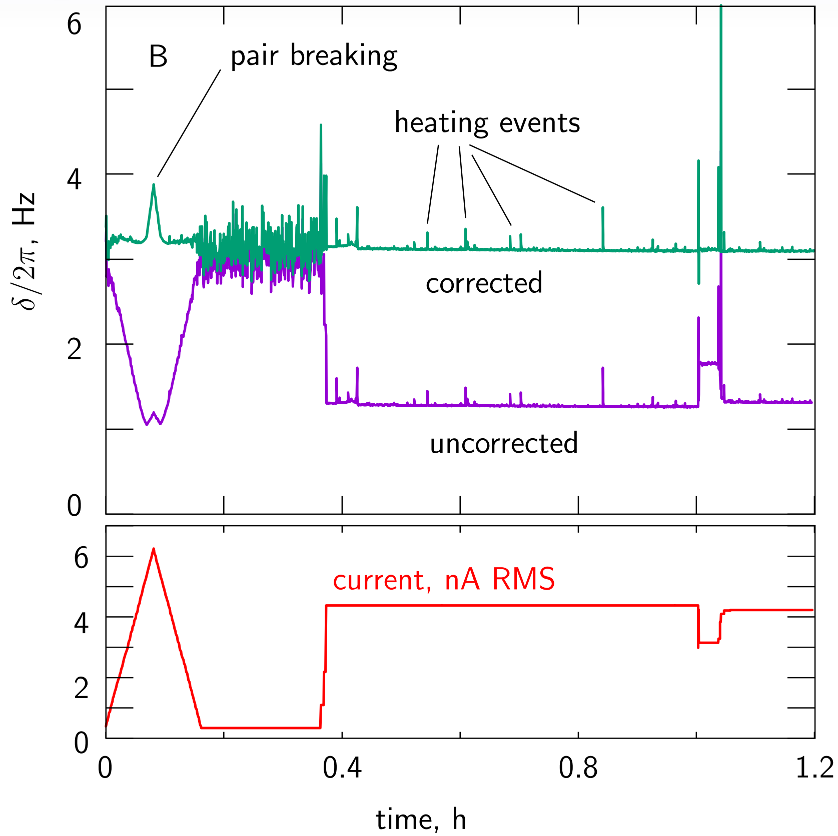
To process thermometry data we need function S(v).
It depends on pressure and temperature (via v_0) ,
on wire intrinsic damping (which depends on magnetic field).

Simultaneous fit of a few frequency sweeps using function $S(x)$ obtained from the left picture

Processing non-linear data



Fitting frequncy sweeps with and without non-linear correction



Applying non-linear correction to tracking mode

Converting resonance width to temperature

Knowledge of function S can give us "zero-velocity" damping δ.
It should be converted to temperature.

Theory:

$$\delta = \frac{p_F^2 v_F N(0)}{\rho_w d_w} \exp\left(-\frac{\Delta}{kT}\right)$$

Pre-factor in this formula can be affected by multiple things:
- type of scattering and surface of the wire
- finite coherence length in ^{3}He (for small wire sizes)
- dust on the wire

Possible approach: use 0.127mm tantalum wires for calibrating thin wires:
easier to make and measure wire geometry, less affected by dust, and coherence length effects.

More information on vibrating wire thermometry: arXiv:2303.01189