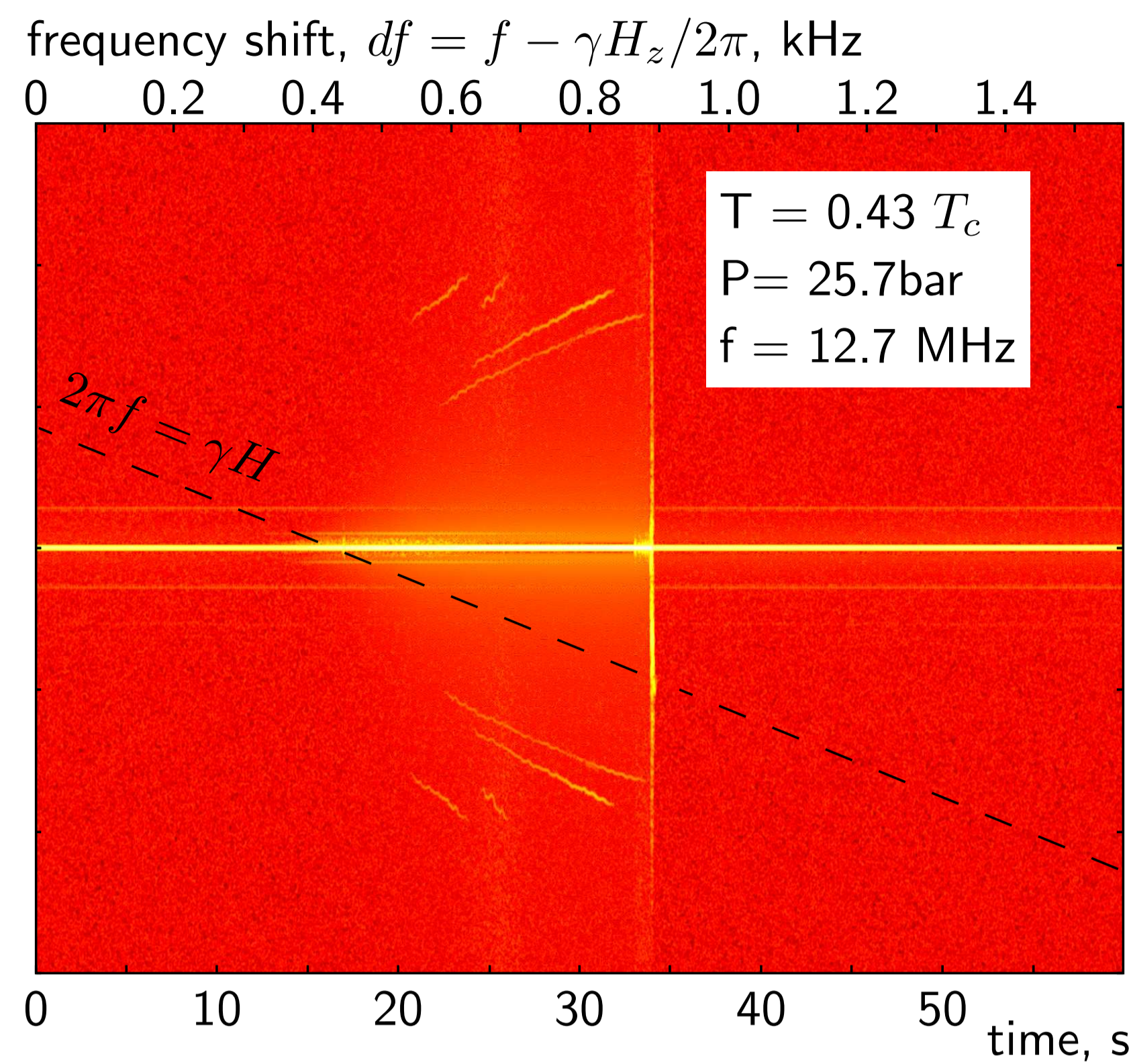
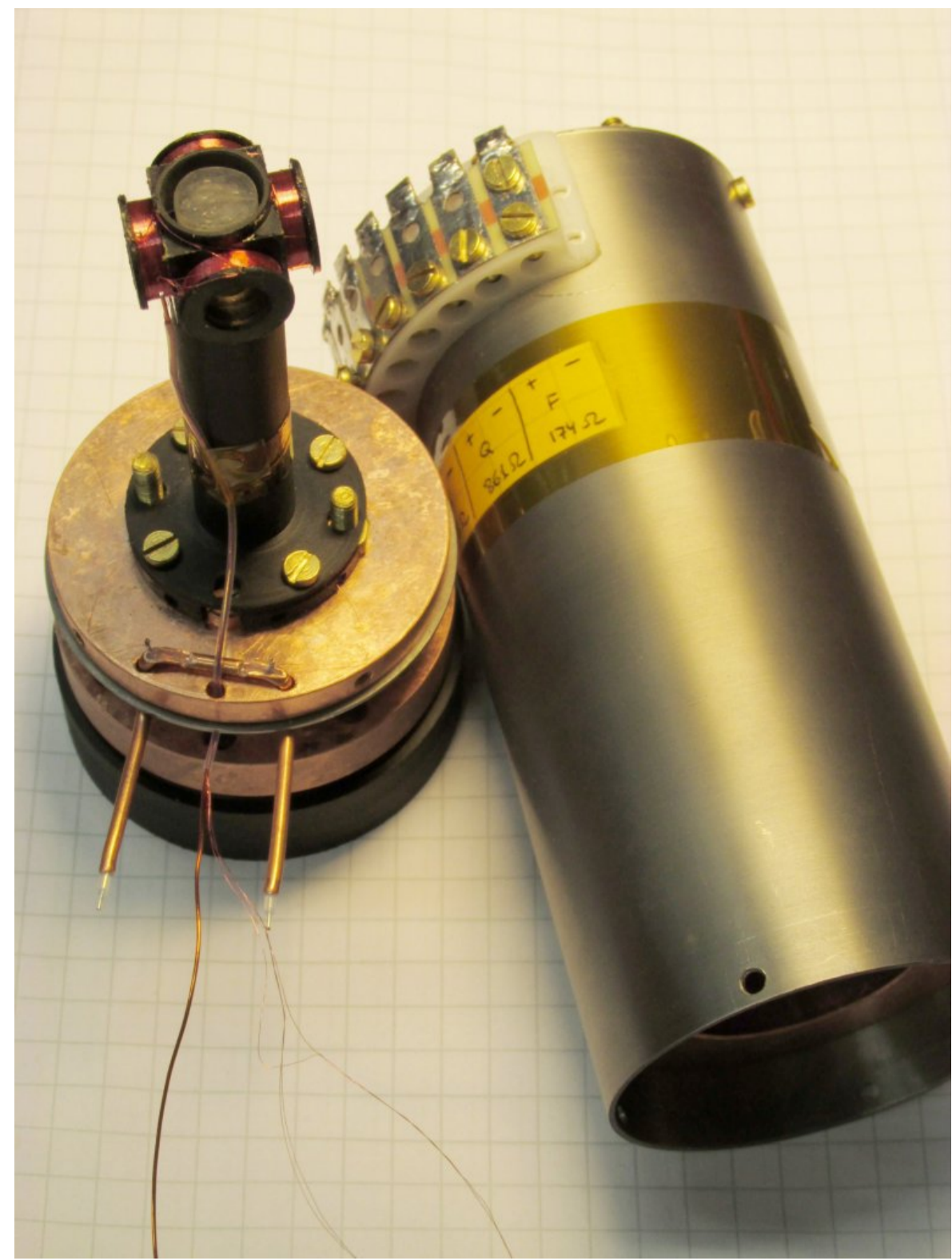


Motivation: Experiment in Helsinki, 2018



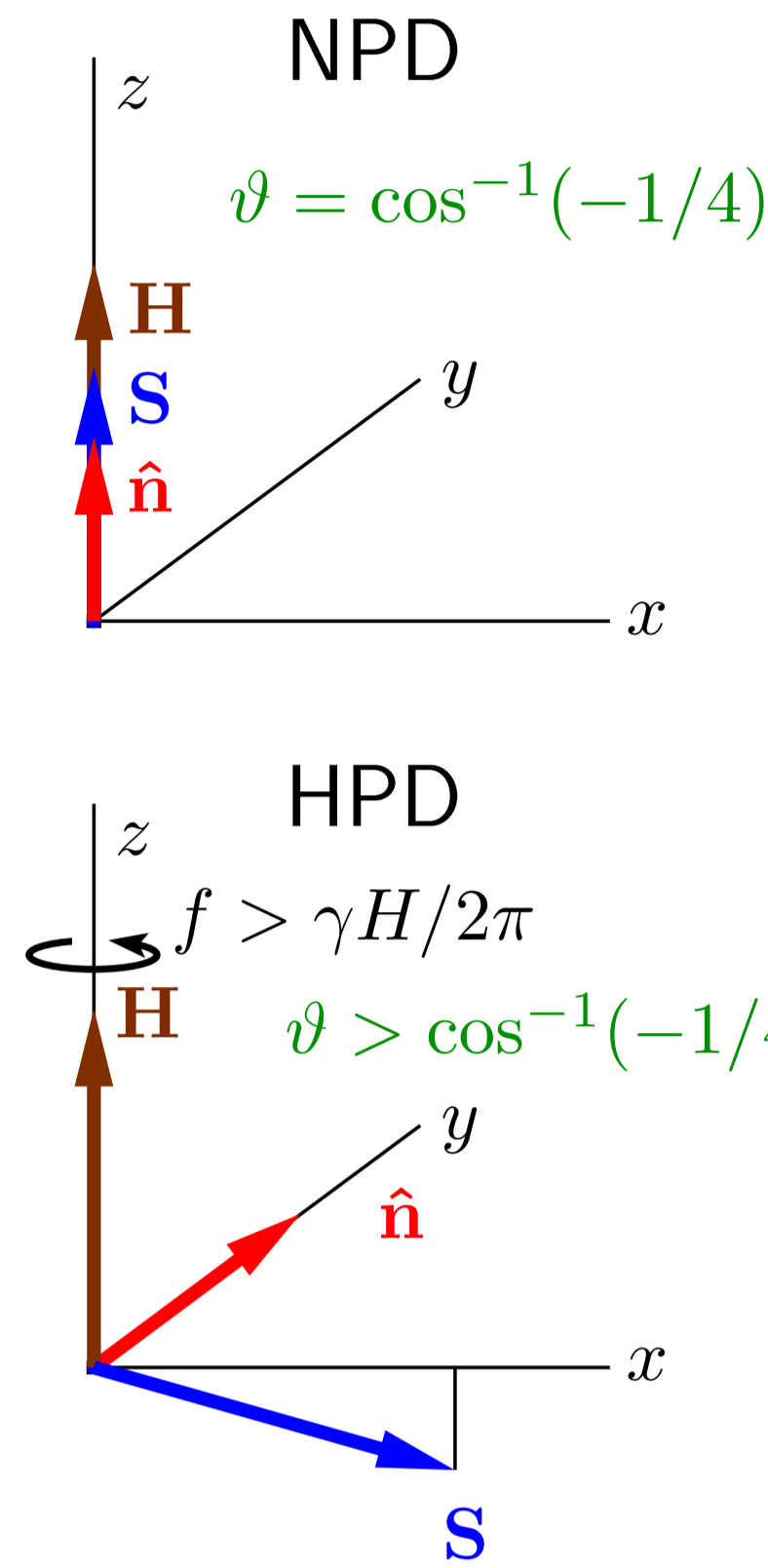
In HPD state we observed many oscillation modes with frequency  $\propto \sqrt{df}$  localized in different parts of the cell. Oscillations of solitons?

Spin dynamics, HPD and NPD states

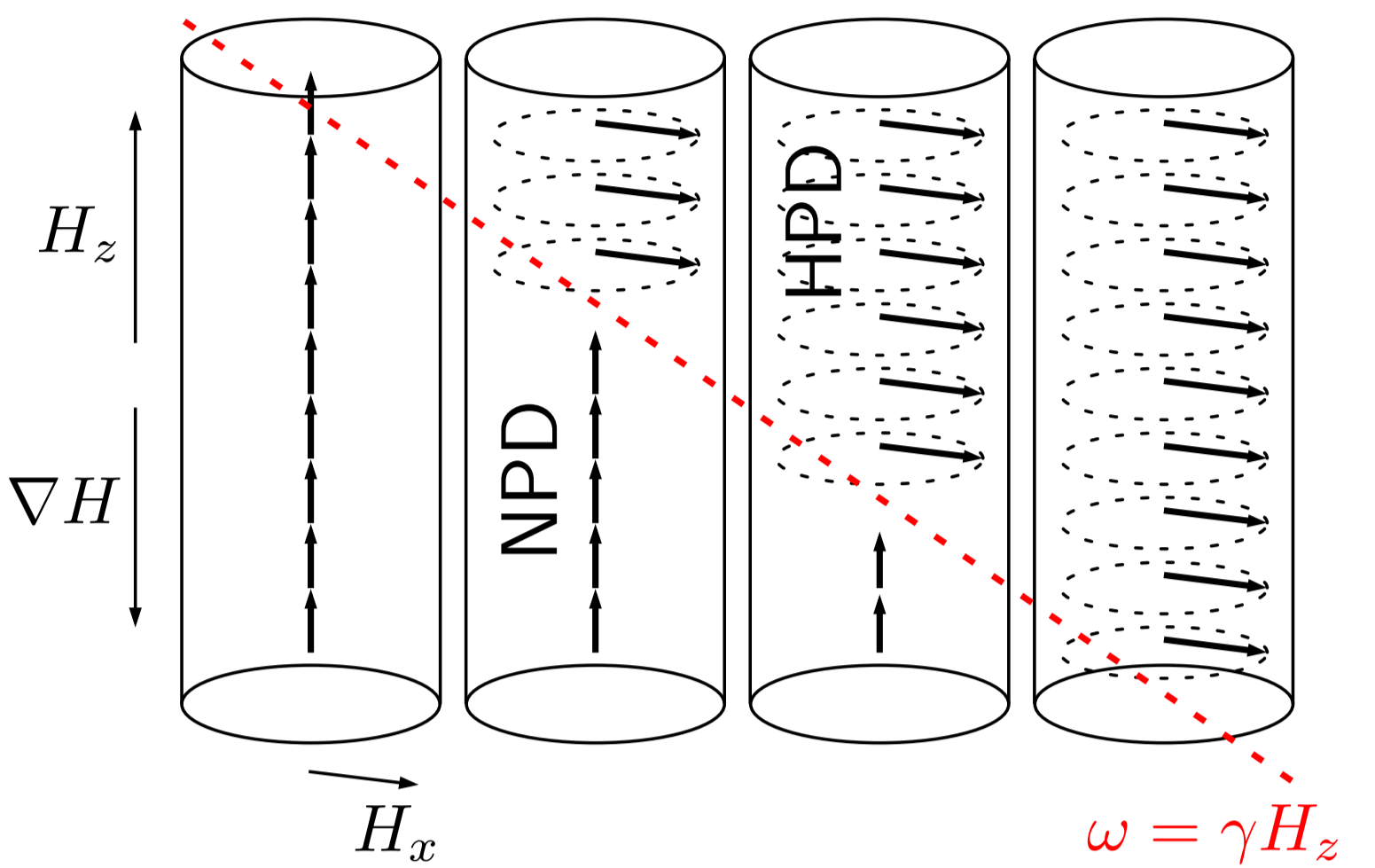
$^3\text{He-B}$ : spin state is described by rotation matrix  $R(\mathbf{n}, \vartheta)$

Leggett equations:

$$\begin{cases} \dot{S}_a = [\mathbf{S} \times \gamma \mathbf{H}]_a + T_a(R) \\ \dot{R}_{aj} = e_{abc} R_{cj} \left( \frac{\gamma^2}{\chi_B} \mathbf{S} - \gamma \mathbf{H} \right)_b \end{cases}$$

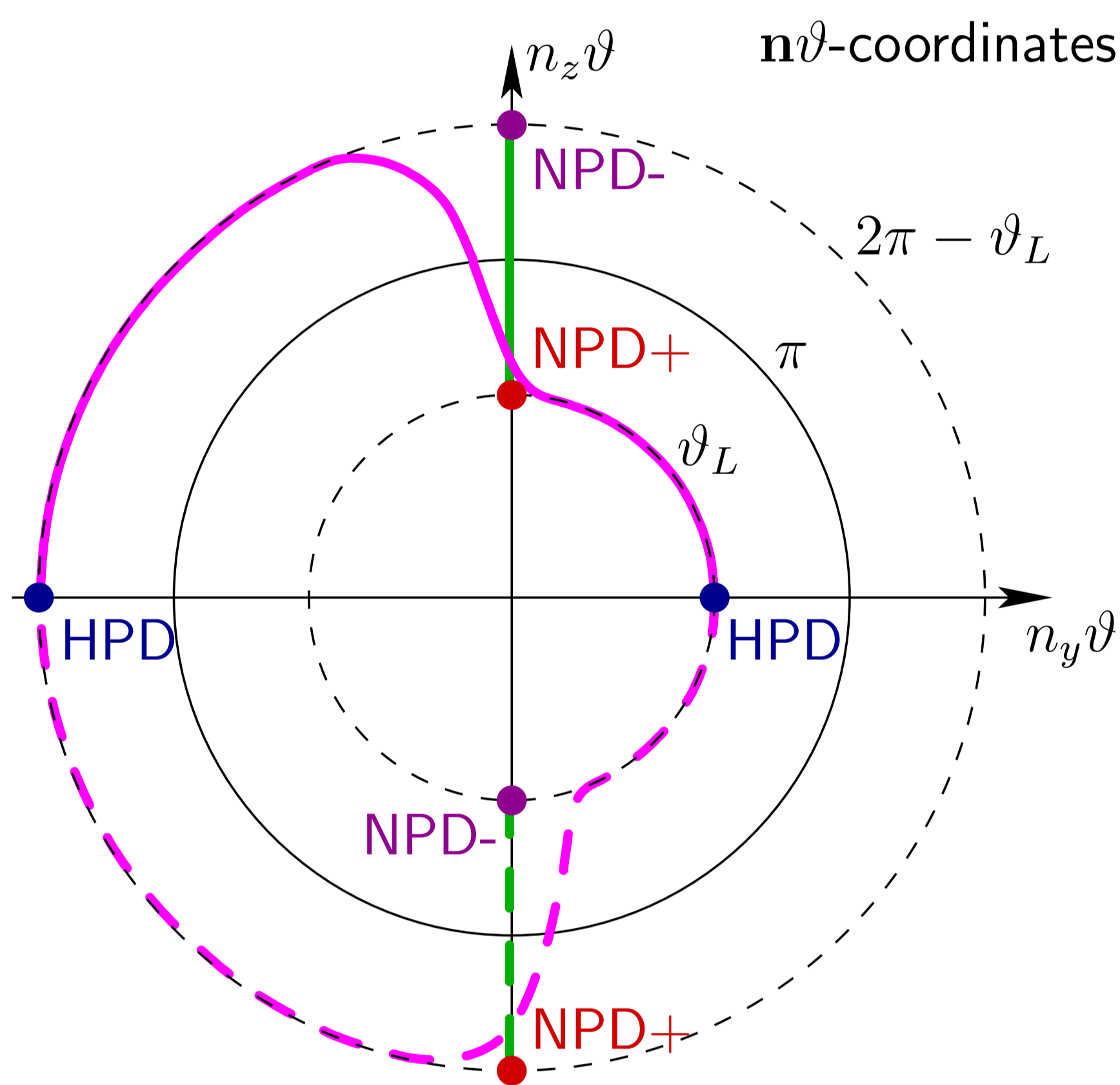


creation of HPD in CW NMR



Frequency shift:  $df = f - \gamma H_z / 2\pi$

$\vartheta$ -solitons



Soliton in NPD: only  $\vartheta$  is changing. Exact solution:

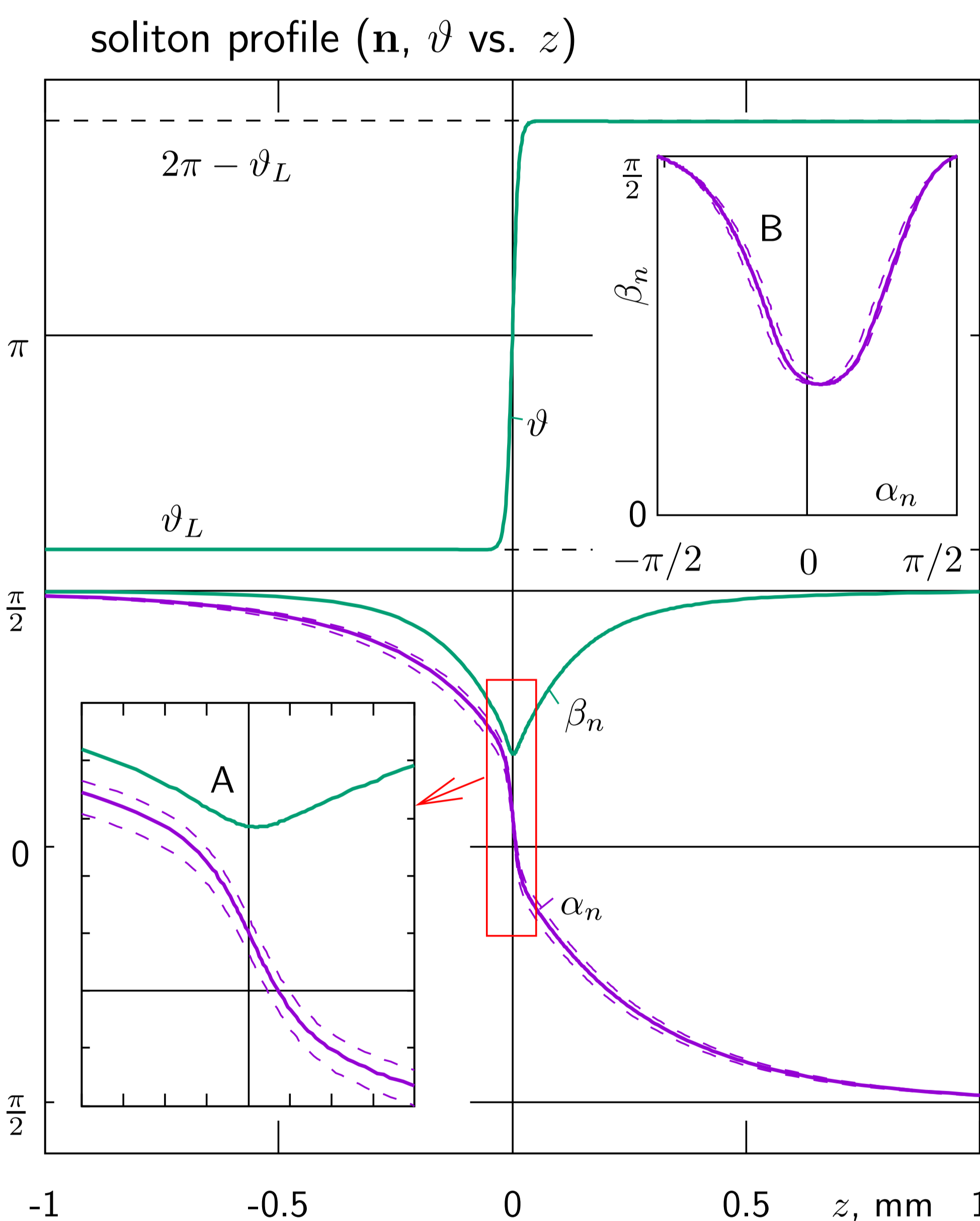
$$\vartheta = 2 \tan^{-1} \left[ \sqrt{\frac{3}{5}} \tanh \left( \sqrt{\frac{65}{64}} \frac{\pm z}{\xi_D} \right) \right] + \pi.$$

$$\xi_D^2 = \frac{65}{16} (2c_{\perp}^2 - c_{\parallel}^2) / \Omega_B^2$$

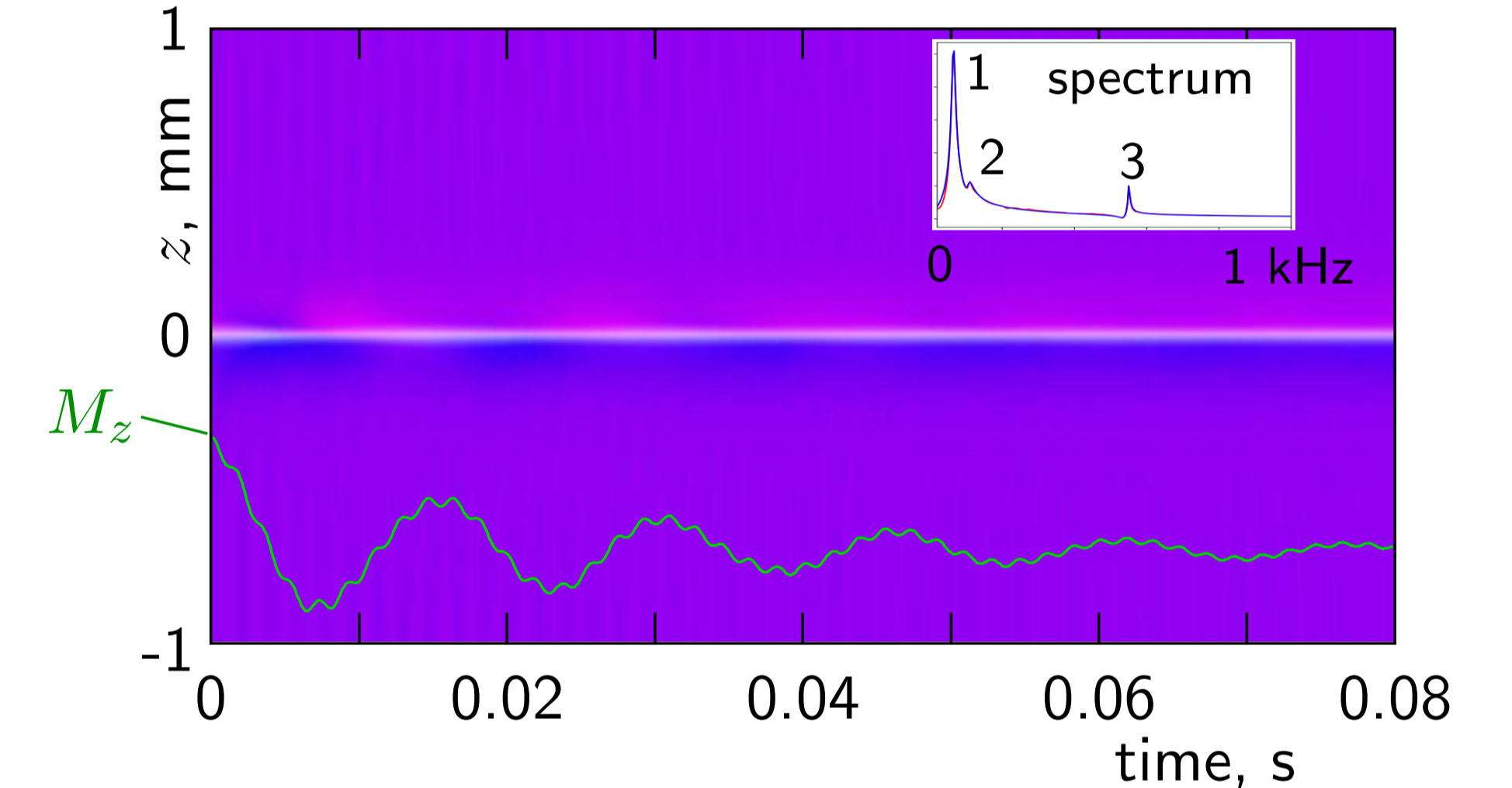
Soliton in HPD: small core where  $\vartheta$  is changing and long tails where  $\mathbf{n}$  is rotating.

Calculation of  $\vartheta$ -soliton in HPD

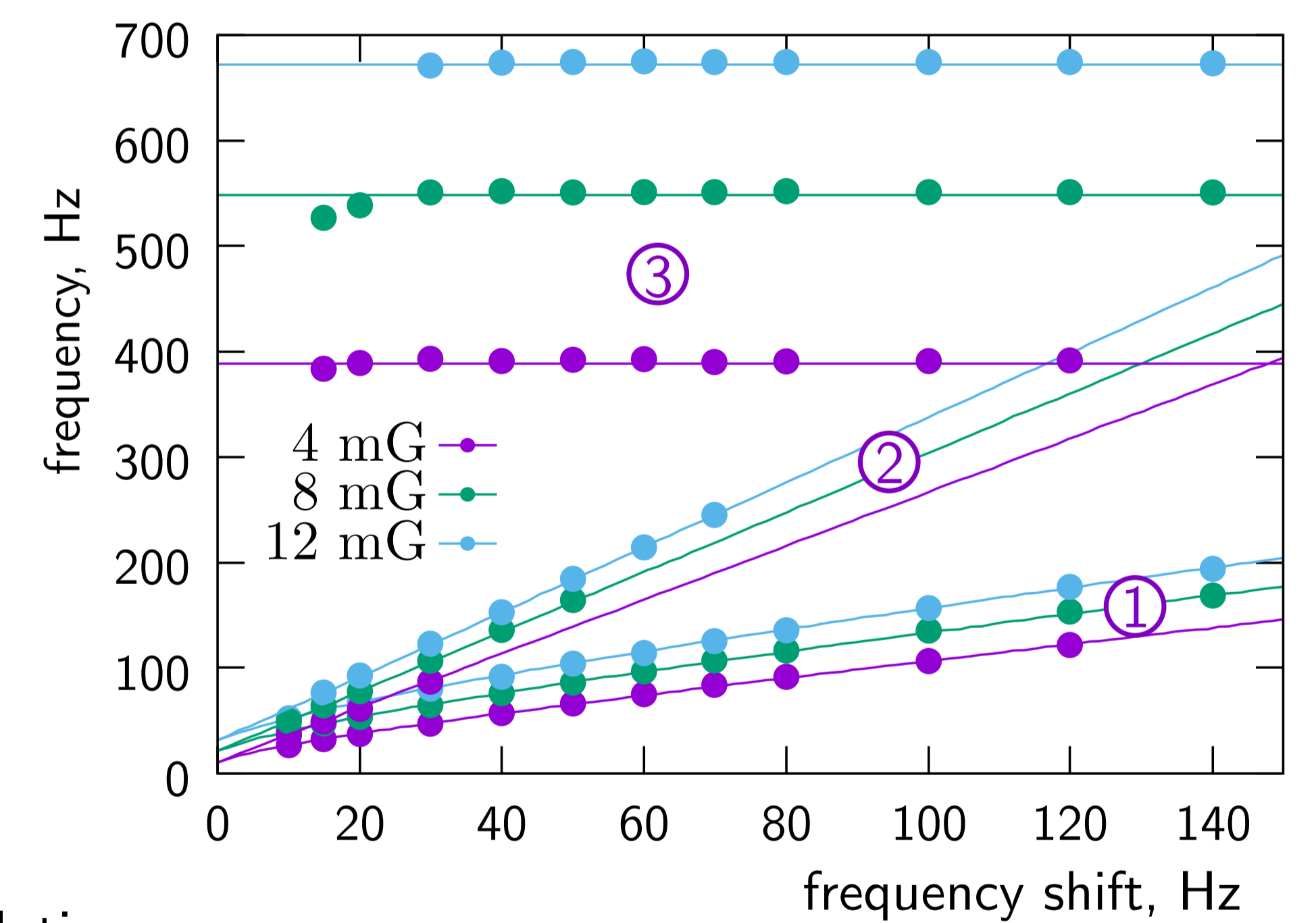
We solve Leggett equations numerically in 1D geometry to get soliton profile and dynamics



soliton oscillations after step change of  $H_z$  color: orientation of  $\mathbf{n}$



Oscillation modes for three values of RF field



Oscillation of soliton membrane

Soliton mass can be calculated from kinetic energy of a moving soliton. Mass density:

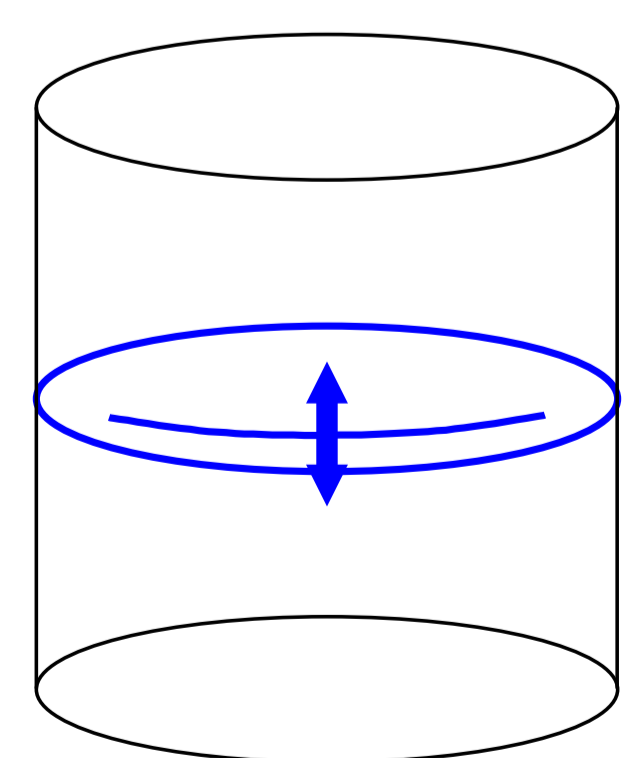
$$m = \frac{\chi_B}{\gamma^2} [(\vartheta')^2 + 2(1 - \cos \vartheta) (\mathbf{n}')^2]$$

Tension: total energy of membrane unit area

For  $\vartheta$ -soliton in NPD:

$$M = \frac{13}{3} \frac{\chi_B}{\gamma^2 \xi_D^2} \int_{-\infty}^{+\infty} (\cos \vartheta(z) + 1/4)^2 dz,$$

$$T = \frac{16}{15} \frac{\chi_B \Omega_B^2}{\gamma^2} \int_{-\infty}^{+\infty} (\cos \vartheta(z) + 1/4)^2 dz.$$



Ratio of tension and mass give square of wave velocity  $C$  along the membrane,

$$C^2 = 2c_{\perp}^2 - c_{\parallel}^2.$$

First oscillation mode:  $f = 2.405 C/r$

① 1st mode of soliton oscillations:

$$f_1^2 \approx 0.75 (\gamma H_x / 2\pi)^2 + 4.0 (\gamma H_x / 2\pi) df \text{ does not depend on } c_{\parallel, \perp} \text{ and } \Omega_B (T \text{ and } P)$$

② second mode of soliton oscillations:

weak, depends on  $T$  and  $P$

③ HPD uniform oscillations:  $f_3^2 = \frac{1}{(2\pi)^2} \frac{4}{\sqrt{15}} \frac{\gamma H_x \gamma H_z \Omega_B^2}{(\gamma H_z)^2 + 8/3 \Omega_B^2}$

Mass profile and difference between HPD and NPD solitons

